Online Appendix for
"Norms and Emotions"

O.A Other Predictions from the Simple Model

In Section 2.1, we present a simple PS model of prosocial behavior and show its insightfulness. In this section, we discuss additional implications of the model, such as conformative versus pride-seeking behavior, and boomerang effects.

Recall that $x = 1$ denotes engaging in prosocial behavior, and $x = 0$ denotes non-engagement. Private and social payoffs are in conflict: $u(0) = \bar{u} > 0 = u(1)$ and $w(0) = 0 < \bar{w} = w(1)$, with $\beta\bar{w} < \bar{u} < \bar{w}$. At menu $\{0, 1\}$, the DM chooses an action by comparing the ex post utility of action 0, $U(0; \{0, 1\}) = \bar{u} - w(\varphi_r(\{0, 1\}))$, with that of action 1, $U(1; \{0, 1\}) = \beta\left[\bar{w} - w(\varphi_r(\{0, 1\}))\right]$. We compare decisions in the benchmark case $r(0) > r(1)$ (prosocial behavior is perceived as uncommon) with those in the post-intervention case $r'(0) < r'(1)$ (prosocial behavior is perceived as common).

Conformity and pride seeking. The DM conforms to the reference alternative both in the benchmark case ($x = \varphi_r(\{0, 1\}) = 0$) and post-intervention case ($x = \varphi_{r'}(\{0, 1\}) = 1$). By contrast, if we modify the benchmark assumption so that $\beta\bar{w} > \bar{u}$, then the DM engages in prosocial behavior under both scenarios. In the modified benchmark, the DM deviates from the reference to seek pride. Thus, our model can produce conformative or pride-seeking behavior depending on β. Typical empirical findings suggest β is small (see footnote 14); still, in some contexts, individuals may seek to perform better than a natural reference point.

Boomerang effect. In a field experiment on electricity consumption, Schultz et al. (2007) find that providing descriptive information on neighbors' electricity usage led to desired electricity saving by high-consuming households but increased consumption by low-consuming households. To explain the latter result (which Schultz et al. (2007) call a "boomerang effect") without complicating the model, let $x = 0$ and $x = 1$ denote high consumption and low consumption of electricity, respectively, and suppose that the low-consuming households originally perceive norms (w, r') but the intervention updates the perceptions to (w, r). By the analysis in Section 2.1, the low-consuming households originally choose $x = 1$ but the intervention causes them to switch to $x = 0$. Thus, our model can explain the boomerang effect by a shift of the perceived descriptive norm toward higher consumption.

The purpose of the above example is to illustrate the importance of considering the perceived norms of individuals when introducing a policy, rather than develop a more thorough model. For example, the reduction in the electricity consumption by high-consuming households can be explained by the opposite shift in the perceived descriptive norm. Instead of developing a model which accommodates both types of households (possibly requiring more

than two options), we note that even the direction of a policy effect, as well as its magnitude, crucially depends on what norms the households perceive prior to the intervention.

O.B   Supplemental Proofs

O.B.1   Supplemental Proofs for Theorem 1 (Sufficiency Part)

O.B.1.1   Supplemental Results for Lemma 1

Lemma O1. *Suppose Axioms 4 and 5 hold. Then, for any $a, b, c \in \Delta$ and $\alpha \in (0, 1)$, $a \succ^* b$ implies $a\alpha c \succ^* b\alpha c$.*

Proof. By definition, we have $A \cup \{a\} \succ A$ and $a \notin \mathcal{C}(A \cup \{a\})$ for some $A \ni b$. Then, mixing each term with $\{c\}$ with mixing rate α yields the result.   □

Lemma O2. *Suppose Axioms 1-3 hold. Then, the following statements hold.*

(i) *If $a \succ^* b$, then for any $c \in \Delta$, there exists $\alpha^* \in (0, 1)$ such that $a\alpha c \succ^* b$ for all $\alpha \in (\alpha^*, 1)$.*

(ii) *If $b \succ^* c$, then for any $a \in \Delta$, there exists $\beta^* \in (0, 1)$ such that $b \succ^* c\beta a$ for all $\beta \in (\beta^*, 1)$.*

Proof. (i) By $a \succ^* b$, we have $A \cup \{a\} \succ A$ and $a \notin \mathcal{C}(A \cup \{a\})$ for some $A \ni b$. By Axiom 2(i), there exists $\alpha_1 \in (0, 1)$ such that $A \cup \{a\alpha c\} \succ A$ for all $\alpha \in (\alpha_1, 1)$. By Axiom 3(iii-b), we have $\alpha_2 \in (0, 1)$ such that $a\alpha c \notin \mathcal{C}(A \cup \{a\alpha c\})$ for all $\alpha \in (\alpha_2, 1)$. Thus, $\alpha^* \equiv \max\{\alpha_1, \alpha_2\}$ has the desired property.

(ii) By $b \succ^* c$, we have $B \cup \{b\} \succ B$ and $b \notin \mathcal{C}(B \cup \{b\})$ for some $B \ni c$. By Axioms 2(i) and 3(iii), there exist $\beta_1, \beta_2 \in (0, 1)$ such that $[B\beta\{a\}] \cup \{b\} \succ B\beta\{a\}$ for all $\beta \in (\beta_1, 1)$ and $b \notin \mathcal{C}([B\beta\{a\}] \cup \{b\})$ for all $\beta \in (\beta_2, 1)$.[1] Thus, $b \succ^* c\beta a$ for all $\beta > \beta^* \equiv \max\{\beta_1, \beta_2\}$.   □

Lemma O3. *Suppose Axioms 1-5 hold. If $a \succ^* b$ holds, then there exists $c \in \text{int}(\Delta)$ such that $c \not\succ^* a$ and $c \succ^* b$.*

Proof. Suppose $a \succ^* b$. By Lemma O1, we have $a \succ^* a.5b \succ^* b$. If $a.5b \in \text{int}(\Delta)$, then Axiom 3(i) implies $c = a.5b$ has the desired property. Otherwise, take any $\alpha \in (0, 1)$ and $d \in \text{int}(\Delta)$, and let $c = (a.5b)\alpha d \in \text{int}(\Delta)$. Then, by Lemma O2, we have $a \succ^* c \succ^* b$ for α sufficiently close to one.   □

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[1]To show that the former property holds for all sufficiently large $\beta < 1$, note first that Axioms 2(i) and 3(iii-a) ensure $B \cup \{b\} \succ \tilde{B} \succ B$ where $\tilde{B} = \tilde{B}(\gamma) = [B \cup \{b\}]\gamma B$ for some $\gamma \in (0, 1)$. (Otherwise, $\Gamma^L = \{\gamma \in [0, 1] : B \succeq \tilde{B}(\gamma)\}$ and $\Gamma^U = \{\gamma \in [0, 1] : \tilde{B}(\gamma) \succeq B \cup \{b\}\}$ are nonempty closed sets such that $\Gamma^L \cup \Gamma^U = [0, 1]$, so $\tilde{B}(\gamma) \succ \tilde{B}(\gamma)$ for $\gamma \in \Gamma^L \cap \Gamma^U$, a contradiction.) Then, by Axioms 2(i) and 3(iii-a), for all sufficiently large $\beta < 1$, we must have $[B\beta\{a\}] \cup \{b\} \succ \tilde{B} \succ B\beta\{a\}$.

Lemma O4. *If Axioms 1-5 hold, then $\succ_r$ is transitive.*[2]

Proof. Suppose $a \succ_r b \succ_r c$. By Lemma O3, we have some $d \in \text{int}(\Delta)$ such that $d \not\succ^* a$ and $d \succ^* b$. If $d \not\succ^* c$, then $c \succ_r b$, contradicting Axiom 3(i). Therefore, $d \succ^* c$, hence $a \succ_r c$. $\quad\square$

O.B.1.2 Supplemental Results for Lemma 2

Below, we impose Axioms 1-5, so $\succeq_r$ admits a linear representation r (Lemma 1).

Lemma O5. *Suppose Axioms 1-5 hold. Then, for any finite $A \in \mathcal{A}$, there exists $a^* \in A$ such that $a^* \succeq_r a$ and $a^* \succeq_w a$ for all $a \in A$.*

Proof. Because $\succeq_w$ is complete by definition and transitive on $\varphi_r(A)$ by Axiom 3(ii), there exists a^* which maximizes $\succeq_w$ on $\varphi_r(A)$. By $a^* \in \varphi_r(A)$, we must have $a^* \succeq_r a$ for all $a \in A$. Also, for any $a \in A \setminus \varphi_r(A)$, we have $a^* \succ_r a$, so Axiom 3(i) implies $a^* \succeq_w a$. Thus, $a^* \succeq_r a$ and $a^* \succeq_w a$ for all $a \in A$. $\quad\square$

Lemma O6. *Suppose that Axioms 1-6 hold and that A and B are finite.*

   (i) If $A \succ A \cup B$, there is $b \in B \setminus A$ such that $b \succ_r a$ or $b \succ_w a$ for all $a \in A$.

   (ii) If $A \cup B \succ A$ and $\mathcal{C}(A \cup B) \cap A \neq \emptyset$, there is $b \in B \setminus A$ such that $b \succ_r a$ for all $a \in A$.

Proof. Note that by the hypotheses, $A \not\supseteq B$ holds for (i)(ii).

   (i) By Lemma O5, there exists $a^* \in A$ such that $a^* \succeq_r a$ and $a^* \succeq_w a$ for all $a \in A$. To prove the contrapositive, suppose that for any $b \in B \setminus A$, there exists $a \in A$ such that $a \succeq_r b$ and $a \succeq_w b$. By transitivity and Axiom 3(i)(ii), $a^* \succeq_r c$ and $a^* \succeq_w c$ for all $c \in A \cup B$. Thus, Axiom 6(i) yields $A \cup B \succeq A$.

   (ii) If the conclusion is false, then we have $a^* \in A$ such that $a^* \succeq_r c$ for all $c \in A \cup B$ and $a^* \succeq_w c$ for all $c \in A$. If $a^* \succeq_w b$ for all $b \in B$, then by Axiom 6(i), it is impossible to have $A \cup B \succ A$ and $\mathcal{C}(A \cup B) \cap A \neq \emptyset$ simultaneously. If $b \succ_w a^*$ for some $b \in B$, then Axiom 6(ii) yields the same conclusion. $\quad\square$

O.B.1.3 Supplemental Results for Lemma 4.

Lemma O7. *Suppose Axioms 3(i) and 6(i) hold. Then, $b \in \mathcal{P}(a) \cup \mathcal{S}(a) \cup \mathcal{N}_1(a)$ implies $\{a, b\} \succ \{a\}$, and $b \in \mathcal{N}_2(a)$ implies $\{a, b\} \sim \{a\}$.*

Proof. If $a \succ_r b$, Axiom 3(i) implies $a \succeq_w b$. Then, Axiom 6(i) yields the conclusion. $\quad\square$

Lemma O8. *Suppose Axioms 1-6 hold. If $\{c, d\} \in \mathcal{B}_S$, then $\{c\} \succeq \{c, d\}$ or $\{d\} \succeq \{c, d\}$.*

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[2] In fact, Axioms 3(i)(ii) are enough to show the transitivity of $\succ_r$, with a longer proof.

Proof. The conclusion trivially holds if $c = d$, so we assume $c \neq d$. Without loss of generality, let $c \succ_r d$. If $d \in \mathcal{S}(c) \cup \mathcal{N}_1(c)$, then $\{d\} \succeq \{c, d\}$ by definition. If $d \in \mathcal{I}(c)$, then Lemma 2(ii) implies either $\{c\} \succeq \{c, d\}$ or $\{d\} \succeq \{c, d\}$. □

Lemma O9. *Suppose Axioms 1-6 hold. If $A \in \mathcal{A}_S \cup \mathcal{A}_N$, then $A \sim \{e\}$ for some $e \in \Delta$.*

Proof. If $A \in \mathcal{A}_N$, then the conclusion follows from Lemma O7. Suppose $A \in \mathcal{A}_S$. Note that iteratively applying Lemma 2 yields $A \succeq \{a\}$ for some $a \in A$.[3] If $A \sim \{a\}$ for some $a \in A$, the conclusion holds. Next, suppose $\{a'\} \succ A \succ \{a\}$ for some $a, a' \in A$. Then, because $\{a\}, \{a'\}, A \in \mathcal{A}_S$, we have $V^S(\{a'\}) > V^S(A) > V^S(\{a\})$. By linearity, there exists $\alpha \in (0, 1)$ such that $V^S(\{a'\alpha a\}) = \alpha V^S(\{a'\}) + (1 - \alpha)V^S(\{a\}) = V^S(A)$. Thus, $\{a'\alpha a\} \sim A$. Finally, to see that $A \succ \{a\}$ for all $a \in A$ does not occur, recall we can write $A = \sum_{m=1}^{M_A} \alpha_m \{a_{1m}, a_{2m}\}$ where $\{a_{1m}, a_{2m}\} \in \mathcal{B}_S$ and $\sum_{m=1}^{M_A} \alpha_m = 1$. By Lemma O8, there exist $(e_m)_{m=1}^{M_A}$, with $e_m \in \{a_{1m}, a_{2m}\}$ for each m, such that $\{e_m\} \succeq \{a_{1m}, a_{2m}\}$. By Axiom 4, we have $\{\sum_{m=1}^{M_A} \alpha_m e_m\} \succeq A$. □

O.B.1.4 Supplemental Results for Theorem 1 (Sufficiency), Continued

Proof of Lemma 6. (i) By Axiom 5(i), $\mathcal{C}(\{a, b\} \alpha \{c, d\}) = \{b\alpha d\}$. Also, by the linearity of r and Axiom 3(i), we have $r(a\alpha c) > r(z)$, hence $a\alpha c \succeq_w z$, for all $z \in A \setminus \{a\alpha c\}$. Therefore, Axiom 6(i) implies $A \sim \{a\alpha c, b\alpha d\}$.

(ii) The same argument as (i) yields the result.

(iii) Let V_{PS} be a function that represents $\succeq$ over finite menus in $\mathcal{A}$. By Eq.(11), there exists $z \in A$ such that $V_{PS}(A) = \min_{z' \in \varphi_r(A)} V_{PS}(\{z, z'\})$. If $z = a\alpha c$, then by $a\alpha c \in \varphi_r(A)$ and Axioms 3(i) and 6(i), we have $V_{PS}(A) \leq V_{PS}(\{a\alpha c\}) < \alpha V_{PS}(\{a, b\}) + (1 - \alpha)V_{PS}(\{c, d\}) = V_{PS}(A)$, a contradiction. If $z = a\alpha d$, then $V_{PS}(A) \leq V_{PS}(\{a\} \alpha \{c, d\}) < V_{PS}(A)$, a contradiction. A similar contradiction results if $z = b\alpha c$. Thus, $V_{PS}(A) = \min_{z' \in \varphi_r(A)} V_{PS}(\{b\alpha d, z'\}) \leq V_{PS}(\{a\alpha c, b\alpha d\})$. Now, suppose $V_{PS}(b) > V_{PS}(\{a, b\})$. Note we have $V_{PS}(A) = V_{PS}(\{b\alpha d, a\alpha c\})$ or $V_{PS}(A) = V_{PS}(\{b\alpha d, b\alpha c\})$. In the latter case, $V_{PS}(A) = \alpha V_{PS}(\{b\}) + (1 - \alpha)V_{PS}(\{c, d\}) > V_{PS}(A)$, a contradiction. Thus, $A \sim \{a\alpha c, b\alpha d\}$.

(iv) Note we have $\varphi_r(A) = \{a\alpha c, b\alpha c\}$. Also, by Axiom 4(iii), we have $\{b\alpha c\} \succ \{a\alpha c, b\alpha c\}$, so $a\alpha c \succ_w b\alpha c$. By Axiom 3(i), $a\alpha c \succeq_w z$ for all $z \in A$. Also, by Axiom 5(i), $\mathcal{C}(A) = \mathcal{C}(\{a, b\}) \alpha \{d\}$. Therefore, Axiom 6(i) yields the desired conclusion.

(v) Let $\mathcal{C}(\{a, b\}) = \{a\}$. We first prove the last two relations in (v-a). By Axiom 5(i), $\mathcal{C}(A) = \{a\alpha d\}$ and $\mathcal{C}(\{a, b\} \alpha \{c\}) = \{a\alpha c\}$. Also, Axiom 4 implies $\{b\alpha c\} \sim \{a\alpha c, b\alpha c\}$,

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[3]Denote $A = \{a_1, \cdots, a_{|A|}\}$ where $\{a_1\} \succeq \{a_2\} \succeq \cdots \succeq \{a_{|A|}\}$. If $a_{|A|-1} \succ_r a_{|A|}$, then Lemma 2(i) implies $\{a_{|A|-1}, a_{|A|}\} \succeq \{a_{|A|-1}\} \succeq \{a_{|A|}\}$. If $a_{|A|} \succ_r a_{|A|-1}$, then Lemma 2(i) implies $\{a_{|A|-1}, a_{|A|}\} \succeq \{a_{|A|}\}$. If $a_{|A|-1} \sim_r a_{|A|}$, then Lemma 2(ii) implies $\{a_{|A|-1}, a_{|A|}\} \succeq \{a_{|A|}\}$. Repeating similar arguments yields $A \succeq \{a_{|A|}\}$.

so $a\alpha c \succ_w b\alpha c$. Also, we have $a\alpha c \succ_r a\alpha d, b\alpha d$, so Axiom 3(i) implies $a\alpha c \succeq_r z$ and $a\alpha c \succeq_w z$ for all $z \in A$. By Axiom 6(i), $A \sim \{a\alpha c, a\alpha d\} \succ \{a\alpha c, b\alpha d\}$. Next, to show the first relation in (v-a), note that $b\alpha c \succeq_r z$ for all $z \in A$, $b\alpha c \succeq_w a\alpha d$ (by Axiom 3(i)), and $a\alpha c \succ_w b\alpha c$. Thus, applying Axiom 6(ii) to $\tilde{A} = \{b\alpha c, a\alpha d\}$ and $\tilde{B} = \{a\alpha c, b\alpha d\}$, we obtain $\{b\alpha c, a\alpha d\} \succ \tilde{A} \cup \tilde{B} = A$. Finally, to show (v-b), suppose $\mathcal{C}(\{a, b\}) = \{a, b\}$. Then we have $\mathcal{C}(A) = \{a\alpha d, b\alpha d\}$, and $a\alpha c, b\alpha c \succeq_r z$ and $a\alpha c, b\alpha c \succeq_w z$ for all $z \in A$. Thus, applying Axiom 6(i) to $\tilde{A} = \{a\alpha c, b\alpha d\}$ and $\tilde{B} = \{b\alpha c, a\alpha d\}$ yields $A \sim \{a\alpha c, b\alpha d\}$ and applying it to $\tilde{A} = \{a\alpha c, a\alpha d\}$ and $\tilde{B} = \{b\alpha c, b\alpha d\}$ yields $A \sim \{a\alpha c, a\alpha d\}$. $\square$

Lemma O10. *Suppose Axioms 1-5 hold, $y \in \mathcal{P}(x)$, and $y' \in \mathcal{S}(x)$.*

(i) There exists $\delta \in (0, 1)$ such that $y(1 - \delta)c \in \mathcal{P}(x)$ and $y'(1 - \delta)c \in \mathcal{S}(x)$ for all $c \in \Delta$.

(ii) $y(1 - \delta)c \in \mathcal{P}(x(1 - \delta)c)$ and $y'(1 - \delta)c \in \mathcal{S}(x(1 - \delta)c)$ for all $c \in \Delta$ and all $\delta \in (0, 1)$.

Proof. (i) By definition, $\{x, y\} \succ \{y\}$ and $\mathcal{C}(\{x, y\}) = \{y\}$. Because the restriction of $\succeq$ to singleton sets is continuous, and because Δ is compact, there exists $\delta_1 \in (0, 1)$ such that $\{x, y(1 - \delta)c\} \succ \{y(1 - \delta)c\}$ for all $c \in \Delta$ and $\delta \in (0, \delta_1)$.[4] Also, by Axiom 3(iii-b) and compactness, we have some $\delta_2 \in (0, 1)$ such that $\mathcal{C}(\{x, y(1 - \delta)c\}) = \{y(1 - \delta)c\}$ for all $c \in \Delta$ and $\delta \in (0, \delta_2)$. Therefore, by taking $\underline{\delta}^P = \min\{\delta_1, \delta_2\}$, the first half of the statement holds for all $\delta < \underline{\delta}^P$. An analogous argument yields $\underline{\delta}^S$ such that the second half of the statement holds for all $\delta < \underline{\delta}^S$. Thus, $\delta < \min\{\underline{\delta}^P, \underline{\delta}^S\}$ satisfies the desired property.

(ii) The conclusion is an immediate consequence of Axioms 4(iii) and 5(iii). $\square$

Proof of Lemma 7. (i) Because $y, c \in \mathcal{P}(x)$, $V_{PS}(\{x, y(1 - \delta)c\}) = V_{PS}(\{x, y\}(1 - \delta)\{x, c\})$ by Lemma 6(i). Therefore, using $\{x, y\}, \{x, c\} \in \mathcal{B}_P$,

$$w_P(c; x, y, \delta) = \frac{1}{\delta}[(1 - \delta)V_{PS}(\{x, y\}) + \delta V_{PS}(\{x, c\}) - (1 - \delta)V_{PS}(\{x, y\}) - \delta V_{PS}(\{c\})]$$
$$= V_{PS}(\{x, c\}) - V_{PS}(\{c\}).$$

(ii) The result follows from $V_{PS}(\{x, y(1 - \delta)x\}) = (1 - \delta)V_{PS}(\{x, y\}) + \delta V_{PS}(\{x\})$.

(iii) By Lemma 6(i), $V_{PS}(\{x, [y(1 - \delta)c]\alpha[y(1 - \delta)c']\}) = V_{PS}(\{x, y(1 - \delta)c\}\alpha\{x, y(1 - \delta)c'\})$. Therefore,

$$w_P(c\alpha c'; x, y, \delta) = \frac{1}{\delta}V_{PS}(\{x, [y(1 - \delta)c]\alpha[y(1 - \delta)c']\}) - \frac{1 - \delta}{\delta}V_{PS}(\{x, y\}) - V_{PS}(\{c\alpha c'\})$$
$$= \frac{\alpha}{\delta}V_{PS}(\{x, y(1 - \delta)c\}) + \frac{1 - \alpha}{\delta}V_{PS}(\{x, y(1 - \delta)c'\})$$
$$- \frac{1 - \delta}{\delta}V_{PS}(\{x, y\}) - \alpha V_{PS}(\{c\}) - (1 - \alpha)V_{PS}(\{c'\})$$

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[4]Let $A = \{x, y\}.5\{y\}$. By Axiom 4, we have $\{x, y\} \succ A \succ \{y\}$. By Axiom 2(i) and the continuity of V_{PS} on singletons, we have $\{x, y(1 - \delta)c\} \succ A$ and $A \succ \{y(1 - \delta)c\}$ for all sufficiently small δ.

$$=\alpha w_P(c; x, y, \delta) + (1-\alpha)w_P(c'; x, y, \delta).$$

(iv) Let $\delta' \in (0, \delta)$. Note that $y(1-\delta')c = y\frac{\delta-\delta'}{\delta}[y(1-\delta)c]$. Because $y, y(1-\delta)c \in \mathcal{P}(x)$, Lemma 6(i) implies $V_{PS}\left(\{x, y\frac{\delta-\delta'}{\delta}[y(1-\delta)c]\}\right) = V_{PS}\left(\{x, y\}\frac{\delta-\delta'}{\delta}\{x, y(1-\delta)c\}\right)$. Therefore, $V_{PS}\left(\{x, y(1-\delta)c\}\right) = \frac{\delta}{\delta'}V_{PS}\left(\{x, y(1-\delta')c\}\right) - \frac{\delta-\delta'}{\delta'}V_{PS}\left(\{x, y\}\right)$. Substituting this into the definition, we have

$$w_P(c; x, y, \delta) = \frac{1}{\delta'}V_{PS}\left(\{x, y(1-\delta')c\}\right) - \frac{\delta-\delta'}{\delta\delta'}V_{PS}\left(\{x, y\}\right) - \frac{1-\delta}{\delta}V_{PS}\left(\{x, y\}\right) - V_{PS}\left(\{c\}\right)$$
$$= w_P(c; x, y, \delta').$$

(v) Our goal is to show $w_P(c; x, y, \delta) = w_P(c; a, b, \delta) + w_P(a; x, y, \delta)$ or, equivalently,

$$\frac{1}{\delta}V_{PS}\left(\{x, y(1-\delta)c\}\right) = \frac{1}{\delta}V_{PS}\left(\{a, b(1-\delta)c\}\right) - \frac{1-\delta}{\delta}V_{PS}\left(\{a, b\}\right)$$
$$- V_{PS}\left(\{a\}\right) + \frac{1}{\delta}V_{PS}\left(\{x, y(1-\delta)a\}\right).$$

By (ii), we have $V_{PS}\left(\{a\}\right) = \frac{1}{\delta}V_{PS}\left(\{a, b(1-\delta)a\}\right) - \frac{1-\delta}{\delta}V_{PS}\left(\{a, b\}\right)$. Substituting this into the above expression, our goal is to show

$$V_{PS}\left(\{x, y(1-\delta)c\}.5\{a, b(1-\delta)a\}\right) = V_{PS}\left(\{a, b(1-\delta)c\}.5\{x, y(1-\delta)a\}\right).$$

Because $y(1-\delta)c, y(1-\delta)a \in \mathcal{P}(x)$ and $b(1-\delta)a, b(1-\delta)c \in \mathcal{P}(a)$, Lemma 6(i) implies that both sides of this equation equal $V_{PS}\left(\{x.5a, [(1-\delta)(y+b)].5[\delta(a+c)]\}\right)$. $\square$

Lemma O11. *Suppose Axioms 1-7 hold, $y(1-\delta)c \in \mathcal{P}(x)$, and $y'(1-\delta)c \in \mathcal{S}(x)$ for all $c \in \Delta$. Then, $\beta \equiv \frac{1}{\alpha} - 1 > 0$, where $\alpha \in (0, 1)$ is as defined in Axiom 7, satisfies the following condition: $w_P(c; x, y, \delta) = \beta w_S(c; x, y', \delta)$ for all $c \in \Delta$.*

Proof. We have

$$\delta\left[w_P(c; x, y, \delta) - \beta w_S(c; x, y', \delta)\right]$$
$$= V_{PS}\left(\{x, y(1-\delta)c\}\right) - (1-\delta)V_{PS}\left(\{x, y\}\right) - \delta V_{PS}\left(\{c\}\right)$$
$$- \beta\left[V_{PS}\left(\{x, y'(1-\delta)c\}\right) - (1-\delta)V_{PS}\left(\{x, y'\}\right) - \delta V_{PS}\left(\{c\}\right)\right]$$
$$= \frac{1}{\alpha}\left[\alpha V_{PS}\left(\{x, y(1-\delta)c\}\right) + (1-\alpha)V_{PS}\left(\{x(1-\delta)c, y'(1-\delta)c\}\right)\right]$$
$$- \frac{1}{\alpha}\left[\alpha V_{PS}\left(\{x(1-\delta)c, y(1-\delta)c\}\right) + (1-\alpha)V_{PS}\left(\{x, y'(1-\delta)c\}\right)\right]$$
$$= \frac{1}{\alpha}\left[\alpha V_{PS}\left(\{x, y(1-\delta)c\}\right) + (1-\alpha)V_{PS}\left(\left\{e^{x(1-\delta)c, y'(1-\delta)c}\right\}\right)\right]$$

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$$-\frac{1}{\alpha}\left[\alpha V_{PS}\left(\{x(1-\delta)c, y(1-\delta)c\}\right) + (1-\alpha)V_{PS}\left(\left\{e^{x,y'(1-\delta)c}\right\}\right)\right]$$
$$= 0$$

where the last equality holds because $y(1-\delta)c \in \mathcal{P}(x) \cap \mathcal{P}(x(1-\delta)c)$ and $y'(1-\delta)c \in \mathcal{S}(x) \cap \mathcal{S}(x(1-\delta)c)$ hold by Lemma O10, so that Axiom 7 applies. Thus, $w_P(c; x, y, \delta) = \beta w_S(c; x, y', \delta)$ where $\beta > 0$. $\qquad\square$

O.B.2 Supplemental Proofs for Theorem 2

Lemma O12. *Suppose the data are generated by a PS preference. If $A \cup \{a\} \succ A$ and $a \notin \mathcal{C}(A \cup \{a\})$, then $\varphi_r(A \cup \{a\}) = \{a\}$ and $w(a) = \max_{c' \in \varphi_r(A \cup \{a\})} w(c') < \max_{c' \in \varphi_r(A)} w(c')$.*

Proof. Suppose $A \cup \{a\} \succ A$ and $a \notin \mathcal{C}(A \cup \{a\})$. If $\max_{c' \in \varphi_r(A \cup \{a\})} w(c') \geq \max_{c' \in \varphi_r(A)} w(c')$, then $V_{PS}(A \cup \{a\}) = G\left(A \cup \{a\}, \max_{c' \in \varphi_r(A \cup \{a\})} w(c')\right) = G\left(A, \max_{c' \in \varphi_r(A \cup \{a\})} w(c')\right) \leq G\left(A, \max_{c' \in \varphi_r(A)} w(c')\right) = V_{PS}(A)$ where the second equality follows from $a \notin \mathcal{C}(A \cup \{a\})$. This is a contradiction. Thus, $\max_{c' \in \varphi_r(A \cup \{a\})} w(c') < \max_{c' \in \varphi_r(A)} w(c')$, and we must have $\varphi_r(A \cup \{a\}) = \{a\}$ and $\max_{c' \in \varphi_r(A \cup \{a\})} w(c') = w(a)$.[5] $\qquad\square$

Lemma O13. *Suppose the data are generated by a weakly nondegenerate PS preference. If $a \succ_w b$, then $r(a) \geq r(b)$ and $w(a) > w(b)$.*

Proof. Consider the following exhaustive cases.

Case 1. If $\{b\} \succ \{a, b\}$, then $g(b, w(b)) = u(b) > \max_{c \in \{a,b\}} g\left(c, \max_{c' \in \varphi_r(\{a,b\})} w(c')\right) \geq g\left(b, \max_{c' \in \varphi_r(\{a,b\})} w(c')\right)$, so $\max_{c' \in \varphi_r(\{a,b\})} w(c') > w(b)$, yielding the conclusion.

Case 2. If $\{b\} \sim \{a, b\}$ and $\mathcal{C}(\{a, b\}) = \{a\}$, then $g(b, w(b)) = u(b) = g\left(a, \max_{c' \in \varphi_r(\{a,b\})} w(c')\right) > g\left(b, \max_{c' \in \varphi_r(\{a,b\})} w(c')\right)$, so the conclusion holds as in *Case 1*.

Case 3. If $a \sim_r b$ and $\{a\} \sim \{a, b\} \succ \{b\}$, then we have $a \in \mathcal{C}(\{a, b\})$ (otherwise, $a \succ_r b$, a contradiction). Thus, $u(a) = g\left(a, \max_{c' \in \{a,b\}} w(c')\right)$. This in turn implies $\max_{c' \in \{a,b\}} w(c') = w(a) \geq w(b)$. By weak nondegeneracy, the straight indifference curves of r and w cross each other (see Lemma 17). Because $r(a) = r(b)$ and $a \neq b$, we have $w(a) > w(b)$. $\qquad\square$

O.B.3 Proof of Theorem 1 (Necessity Part)

Below, we show that a nondegenerate preference that has a PS representation satisfies each axiom. Proofs of Axiom 1 and Axiom 2(iii) are straightforward and omitted.

To proceed to other axioms, note first that the functions $g(c, R) = u(c) - \max\{R - w(c), 0\} + \beta \max\{w(c) - R, 0\}$ and $G(A, R) = \max_{c \in A} g(c, R)$ are strictly decreasing and continuous in R and that G is continuous in A.

---

[5]Note that for any $A, B \in \mathcal{A}$, $\varphi_r(A \cup B) \in \{\varphi_r(A), \varphi_r(B), \varphi_r(A) \cup \varphi_r(B)\}$.

Axiom 2(i). Suppose $A \succeq B_n$ for all n and $B_n \to B$. Then, because $\max_{c' \in \varphi_r(B)} w(c') \geq \lim_{n \to \infty} \max_{c' \in \varphi_r(B_n)} w(c')$, we have

$$
\begin{aligned}
V_{PS}(A) = G\left(A, \max_{c' \in \varphi_r(A)} w(c')\right) &\geq \lim_{n \to \infty} G\left(B_n, \max_{c' \in \varphi_r(B_n)} w(c')\right) \\
&= G\left(B, \lim_{n \to \infty} \max_{c' \in \varphi_r(B_n)} w(c')\right) \\
&\geq G\left(B, \max_{c' \in \varphi_r(B)} w(c')\right) = V_{PS}(B).
\end{aligned}
$$

Axiom 2(ii). Suppose $A \succ B \succ C$. Note

$$
\begin{aligned}
V_{PS}(A\alpha C) &= G\left(A\alpha C, \max_{c' \in \varphi_r(A\alpha C)} w(c')\right) \\
&= G\left(A\alpha C, \alpha \max_{c' \in \varphi_r(A)} w(c') + (1-\alpha) \max_{c' \in \varphi_r(C)} w(c')\right).
\end{aligned}
$$

By the continuity of G, $V_{PS}(A\alpha C) \approx V_{PS}(C) < V_{PS}(B)$ for sufficiently small $\alpha \in (0,1)$.

Axiom 3(i). If $a \succ_r b$, then Theorem 2 implies $r(a) > r(b)$. Thus, Theorem 2 implies $b \not\succ_r a$, and Lemma O13 implies $b \not\succ_w a$. Similarly, if $a \succ_w b$, then Lemma O13 implies $r(a) \geq r(b)$ and $w(a) > w(b)$, so we cannot have $b \succ_r a$ or $b \succ_w a$.

Axiom 3(ii). Suppose $a \succ^* b \succ^* c$. By definition, there exist $A \ni b$ and $B \ni c$ such that $A \cup \{a\} \succ A$, $a \notin \mathcal{C}(A \cup \{a\})$, $B \cup \{b\} \succ B$, and $b \notin \mathcal{C}(B \cup \{b\})$. Now, let $C = A \cup B$. By Lemma O12, we have $\varphi_r(C \cup \{a\}) = \{a\}$ and $w(a) < \max_{c' \in \varphi_r(A)} w(c') = \max_{c' \in \varphi_r(C)} w(c')$.[6] The representation implies $a \notin \arg\max_{d \in C \cup \{a\}} g(d, w(a)) = \mathcal{C}(C \cup \{a\})$ and $C \cup \{a\} \succ C$. Thus, $a \succ^* c$. Next, if $a \sim_r b \sim_r c$, Theorem 2 implies $[a \succeq_w b] \wedge [b \succeq_w c] \Leftrightarrow [w(a) \geq w(b)] \wedge [w(b) \geq w(c)] \Rightarrow w(a) \geq w(c) \Leftrightarrow a \succeq_w c$.

Axiom 3(iii-a). Suppose $A\alpha_n C \succeq B$ for all n and $\alpha_n \to \alpha$. Because

$$
\lim_{n \to \infty} \max_{c' \in \varphi_r(A\alpha_n C)} w(c') = \lim_{n \to \infty} \left[\alpha_n \max_{c' \in \varphi_r(A)} w(c') + (1-\alpha_n) \max_{c' \in \varphi_r(C)} w(c')\right] = \max_{c' \in \varphi_r(A\alpha C)} w(c'),
$$

we have $A\alpha C \succeq B$ as follows:

$$
\begin{aligned}
V_{PS}(A\alpha C) &= G\left(A\alpha C, \max_{c' \in \varphi_r(A\alpha C)} w(c')\right) \\
&= \lim_{n \to \infty} G\left(A\alpha_n C, \max_{c' \in \varphi_r(A\alpha_n C)} w(c')\right) = \lim_{n \to \infty} V_{PS}(A\alpha_n C) \geq V_{PS}(B).
\end{aligned}
$$

---

[6] For any $a' \in \varphi_r(A)$ and $b' \in \varphi_r(B)$, we have $r(a) > r(a') \geq r(b) > r(b')$.

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Axiom 3(iii-b). Suppose $a^* \in A$ is such that $a^* \succ_r a$ for all $a \in A \setminus \{a^*\}$. Take any $(A_n)_n$ and any $(a_n)_n$ such that $A_n \to A$, $a_n \in \mathcal{C}(A_n)$, and $a_n \to a$. By Theorem 2, $r(a^*) > r(a)$ for all $a \in A \setminus \{a^*\}$, so $\lim_{n\to\infty} \max_{c' \in \varphi_r(A_n)} w(c') = w(a^*)$. By continuity, $g\left(c, \max_{c' \in \varphi_r(A_n)} w(c')\right) \to g\left(c, w(a^*)\right)$ for all c. By $a_n \in \mathcal{C}(A_n)$, we have $g\left(a_n, \max_{c' \in \varphi_r(A_n)} w(c')\right) = G(A_n, \max_{c' \in \varphi_r(A_n)} w(c'))$, so letting $n \to \infty$ yields $g\left(a, w(a^*)\right) = G(A, w(a^*))$. Thus, $a \in \mathcal{C}(A)$.

Axiom 3(iv). By Lemma 18, $a\alpha c \succ^* b\alpha c \Rightarrow r(a) > r(b) \Rightarrow a \succ^* b$.

To prove some of the remaining axioms, we use the following result.

Lemma O14. *Suppose the choice data are generated by a PS preference. (i) If $b \in \mathcal{P}(a)$, then $w(a) < w(b)$. (ii) If $b \in \mathcal{S}(a)$, then $w(a) > w(b)$. (iii) If $b \in \mathcal{N}_1(a)$, then $w(a) = w(b)$.*

Proof. (i) By the representation and the definition of $\mathcal{P}(a)$, $g(b, w(a)) > u(b) = g(b, w(b))$, so $w(a) < w(b)$. (ii) If $b \in \mathcal{S}(a)$, we have $g(b, w(a)) < g(b, w(b))$, so $w(a) > w(b)$. (iii) If $b \in \mathcal{N}_1(a)$, we have $g(b, w(a)) = g(b, w(b))$, so $w(a) = w(b)$. $\square$

Proof of Theorem 1 (Necessity), Continued.

Axiom 4(i). We consider the cases where mixed menus are binary; mixtures with a singleton are considered in Axiom 4(iii). Suppose $b \in \mathcal{P}(a) \cup \mathcal{N}_1(a)$, $d \in \mathcal{P}(c) \cup \mathcal{N}_1(c)$, and $f \in \mathcal{P}(e) \cup \mathcal{N}_1(e)$. By Lemma O14, $r(a) > r(b)$, $w(a) \le w(b)$, $r(e) > r(f)$, and $w(e) \le w(f)$. Therefore, for any $\alpha \in (0, 1)$,

$$
\begin{aligned}
&V_{PS}(\{a, b\} \, \alpha \, \{e, f\}) \\
&= \max_{x \in \{a,b\}\alpha\{e,f\}} \left[u(x) + \beta\left(w(x) - w(a\alpha e)\right)\right] \\
&= \alpha \max_{x \in \{a,b\}} \left[u(x) + \beta\left(w(x) - w(a)\right)\right] + (1-\alpha) \max_{x \in \{e,f\}} \left[u(x) + \beta\left(w(x) - w(e)\right)\right] \\
&= \alpha V_{PS}(\{a, b\}) + (1-\alpha) V_{PS}(\{e, f\}).
\end{aligned}
$$

Similarly, $V_{PS}(\{c, d\} \, \alpha \, \{e, f\}) = \alpha V_{PS}(\{c, d\}) + (1-\alpha) V_{PS}(\{e, f\})$. Thus, $\{a, b\} \succ (\succeq) \{c, d\}$ implies $\{a, b\} \, \alpha \, \{e, f\} \succ (\succeq) \{c, d\} \, \alpha \, \{e, f\}$.

Axiom 4(ii). Again, consider the cases where mixed menus are binary. Take any $\{a, b\}, \{e, f\} \in \mathcal{B}_S$ such that $a \ne b$ and $e \ne f$. By Lemma O14, we can assume without loss of generality that $r(a) \ge r(b)$, $w(a) \ge w(b)$, $r(e) \ge r(f)$, and $w(e) \ge w(f)$. Then,

$$
\begin{aligned}
V_{PS}(\{a, b\} \, \alpha \, \{e, f\}) &= \max_{x \in \{a,b\}\alpha\{e,f\}} \left[u(x) + w(x) - w(a\alpha e)\right] \\
&= \alpha V_{PS}(\{a, b\}) + (1-\alpha) V_{PS}(\{e, f\}).
\end{aligned}
$$

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Therefore, the conclusion of Axiom 4(ii) holds.

Axiom 4(iii). Note that for any $x \in A$, we have

$$g\left(x\alpha c, \max_{c' \in \varphi_r(A\alpha\{c\})} w(c')\right)$$

$$= \alpha u(x) + (1-\alpha)u(c) - \alpha \max\left\{\max_{c' \in \varphi_r(A)} w(c') - w(x), 0\right\} + \alpha\beta \max\left\{w(x) - \max_{c' \in \varphi_r(A)} w(c'), 0\right\}$$

$$= \alpha g\left(x, \max_{c' \in \varphi_r(A)} w(c')\right) + (1-\alpha)u(c).$$

Thus, the conclusion follows from

$$V_{PS}\left(A\alpha\{c\}\right) = \max_{x \in A} g\left(x\alpha c, \max_{c' \in \varphi_r(A\alpha\{c\})} w(c')\right)$$

$$= \alpha \max_{x \in A} g\left(x, \max_{c' \in \varphi_r(A)} w(c')\right) + (1-\alpha)u(c) = \alpha V_{PS}(A) + (1-\alpha)V_{PS}(\{c\}).$$

Axiom 5(i). Consider first the case where $b \in \mathcal{P}(a) \cup \mathcal{N}_1(a)$ and $d \in \mathcal{P}(c) \cup \mathcal{N}_1(c)$. Following the proof of Axiom 4(i),

$$\mathcal{C}(\{a,b\}\alpha\{c,d\})$$

$$= \arg\max_{x \in \{a,b\}\alpha\{c,d\}} [u(x) + \beta(w(x) - w(a\alpha c))]$$

$$= \alpha \arg\max_{x \in \{a,b\}} [u(x) + \beta(w(x) - w(a))] + (1-\alpha)\arg\max_{x \in \{c,d\}} [u(x) + \beta(w(x) - w(c))]$$

$$= \mathcal{C}(\{a,b\})\alpha\mathcal{C}(\{c,d\}).$$

Proof for the case $b \in \mathcal{S}(a) \cup \mathcal{N}_1(a) \cup \mathcal{I}(a)$ and $d \in \mathcal{S}(c) \cup \mathcal{N}_1(c) \cup \mathcal{I}(a)$ is analogous: letting $w(a) \geq w(b)$ and $w(c) \geq w(d)$ without loss of generality,

$$\mathcal{C}(\{a,b\}\alpha\{c,d\}) = \arg\max_{x \in \{a,b\}\alpha\{c,d\}} [u(x) + w(x) - w(a\alpha c)] = \mathcal{C}(\{a,b\})\alpha\mathcal{C}(\{c,d\}).$$

Before proving Axiom 5(ii), we note that Axiom 5(iii) can be shown by following the proof of Axiom 4(iii). Thus, the proof of Axiom 5(iii) is omitted.

Axiom 5(ii). For (ii-a), suppose $A = \{a,b\}\alpha\{a,c\}$, $b \in \mathcal{N}_2(a)$, $c \in \mathcal{P}(a)$, $\{a, b\alpha c\} \succeq \{b\}\alpha\{a,c\}$, and $\mathcal{C}(\{a, b\alpha c\}) = \{b\alpha c\}$. By Axiom 5(iii), we have $\mathcal{C}(\{b\}\alpha\{a,c\}) = \{b\alpha c\}$. By the representation, $g(b\alpha c, w(a)) = V_{PS}(\{a, b\alpha c\}) \geq V_{PS}(\{b\}\alpha\{a,c\}) = g(b\alpha c, w(b\alpha a))$. Therefore, we have $w(b\alpha a) \geq w(a)$, so $w(b) \geq w(a)$. Because $r(a) > r(b), r(c)$ and $w(a) \leq w(b), w(c)$, following the proof of Axiom 5(i) yields $\mathcal{C}(A) = \mathcal{C}(\{a,b\})\alpha\mathcal{C}(\{a,c\})$. Proof of

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(ii-b) is analogous, once we note that the assumptions imply $g(b\alpha c, w(b\alpha a)) \geq g(b\alpha c, w(a))$ so that we have $w(a) \geq w(b)$, as well as $r(a) > r(b), r(c)$ and $w(a) > w(c)$.

Axiom 6(i). Suppose there exists $a^* \in A$ such that $a^* \succeq_r c$ and $a^* \succeq_w c$ for all $c \in A \cup B$. Then, by Theorem 2, $\max_{y \in \varphi_r(A \cup B)} w(y) = \max_{y \in \varphi_r(A)} w(y) = w(a^*)$. Therefore, $V_{PS}(A \cup B) = \max_{x \in A \cup B} g(x, w(a^*)) \geq \max_{x \in A} g(x, w(a^*)) = V_{PS}(A)$ and the inequality is strict if and only if $\mathcal{C}(A \cup B) \cap A = \arg \max_{x \in A \cup B} g(x, w(a^*)) \cap A = \emptyset$.

Axiom 6(ii). Suppose there exists $a^* \in A$ such that $a^* \succeq_r c$ for all $c \in A \cup B$ and $a^* \succeq_w a$ for all $a \in A$, and there exists $b^* \in B$ such that $b^* \succ_w a^*$. By Lemma O13 and Theorem 2, $r(b^*) = r(a^*) \geq r(b)$ for all $b \in B$ and $w(b^*) > w(a^*)$. Without loss of generality, let b^* be a maximizer of $\succeq_w$ on $\varphi_r(B)$. Then, by Theorem 2, $\max_{y \in \varphi_r(A \cup B)} w(y) = w(b^*) > w(a^*) = \max_{y \in \varphi_r(A)} w(y)$. Therefore, if there exists $c \in \mathcal{C}(A \cup B) \cap A$, then $V_{PS}(A \cup B) = g(c, w(b^*)) < g(c, w(a^*)) \leq V_{PS}(A)$.

Axiom 7. Take any $a, b, c, d \in \Delta$ such that $c \in \mathcal{P}(a) \cap \mathcal{P}(b)$ and $d \in \mathcal{S}(a) \cap \mathcal{S}(b)$. Then,

$$
\begin{aligned}
V_{PS}(\{a, c\}) + \beta V_{PS}(\{b, d\}) &= u(c) + \beta(w(c) - w(a)) + \beta[u(d) + w(d) - w(b)] \\
&= u(c) + \beta(w(c) - w(b)) + \beta[u(d) + w(d) - w(a)] \\
&= V_{PS}(\{b, c\}) + \beta V_{PS}(\{a, d\}).
\end{aligned}
$$

Therefore, by letting $\alpha = \frac{1}{1+\beta} \in (0, 1)$, and using $\{e^{b,d}\} \sim \{b, d\}$ and $\{e^{a,d}\} \sim \{a, d\}$,

$$
\begin{aligned}
V_{PS}\left(\{a, c\} \alpha \{e^{b,d}\}\right) &= \alpha V_{PS}\left(\{a, c\}\right) + (1 - \alpha)V_{PS}\left(\{e^{b,d}\}\right) \\
&= \alpha V_{PS}\left(\{b, c\}\right) + (1 - \alpha)V_{PS}\left(\{e^{a,d}\}\right) = V_{PS}\left(\{b, c\} \alpha \{e^{a,d}\}\right).
\end{aligned}
$$

Axiom 8. Suppose there exists $a^* \in A \cap B$ such that $a^* \succeq_r c$ and $a^* \succeq_w c$ for all $c \in A \cup B$. By Theorem 2, $\max_{y \in \varphi_r(A)} w(y) = \max_{y \in \varphi_r(B)} w(y) = w(a^*)$. Now, suppose $a, b \in A \cap B$, $a \in \mathcal{C}(A)$, and $b \in \mathcal{C}(B)$. By the representation, $g(a, w(a^*)) = g(b, w(a^*)) \geq g(c, w(a^*))$ for all $c \in A \cup B$. Therefore, $a \in \mathcal{C}(B)$.

O.B.4 Other Proofs

O.B.4.1 Proof of Claim 1

(i) By assumption, there exist $A \in \mathcal{A}$ and $a \in \Delta$ such that $A \cup \{a\} \succ A$ and $a \notin \mathcal{C}(A \cup \{a\})$. By Lemma O12, $\varphi_r(A \cup \{a\}) = \{a\}$ and $w(a) < \max_{c' \in \varphi_r(A)} w(c')$. Thus, for some $b \in A$, we have $r(a) > r(b)$ and $w(a) < w(b)$. (ii) Without loss of generality, assume $\{a\} \succeq \{b\}$ for all $b \in A$. By assumption, $\max_{y \in \varphi_r(A)} w(y) = w(c) > w(a)$ for some $c \in A$ (otherwise, we would have $A \succeq \{a\}$). Therefore, we have $u(a) \geq u(c)$, $w(a) < w(c)$, and $r(a) \leq r(c)$. If we further

had $u(a) = u(c)$, then $A \succeq \{c\} \sim \{a\}$, a contradiction. Thus, $u(a) > u(c)$. $\qquad \square$

O.B.4.2 Proof of Claim 2

(i) If $\{a, b\} \succ \{b\}$ and $\mathcal{C}(\{a, b\}) = \{b\}$, then we must have $\max_{y \in \varphi(\{a,b\})} w(y) = w(a) < w(b)$, so the DM feels pride by choosing b at $\{a, b\}$. Conversely, pride immediately implies $\{a, b\} \succ \{b\}$, and $\{a, b\} \succ \{a\}$ follows from the representation and $a \notin \mathcal{C}(\{a, b\})$. (ii) Similar to (i). $\square$

O.C Graphical Illustrations of Nondegeneracy and $\succ_r$

In this section, we provide graphical illustrations of the nondegeneracy concepts and the elicitation of the reference ranking, with $\dim(Z) = 3$. Figure O1 illustrates the concepts of nondegeneracy and weak nondegeneracy, providing an example to distinguish the two. It also shows why, in Definition 1, $a \succ^* b$ should be defined using a general menu $A \ni b$ and not just $A = \{b\}$, and it presents a graphical illustration of Theorem 2. Figure O2 then demonstrates how Definition 1(ii-b) helps establish $a \succ_r b$ when we cannot establish the relation via condition (ii-a) (i.e., $a \succ^* b$), which occurs when a is on the boundary of Δ.

Figure O1a illustrates nondegeneracy, which requires that there exist $x, y, y' \in \Delta$ such that $y \in \mathcal{P}(x)$ and $y' \in \mathcal{S}(x)$. For $\mathcal{P}(x)$ to be nonempty, we must have some $y \in \Delta$ such that $r(x) > r(y)$, $w(x) < w(y)$, and $g(x, w(x)) < g(y, w(x))$. The first two conditions ensure that the reference point at $\{x, y\}$ is lower than that at $\{y\}$, and adding the third condition ensures that x is not chosen from $\{x, y\}$. Similarly, for $\mathcal{S}(x)$ to be nonempty, we must have $y' \in \Delta$ such that $r(x) > r(y')$, $w(x) > w(y')$, and $g(x, w(x)) < g(y', w(x))$, ensuring that the reference point at $\{x, y'\}$ is higher than that at $\{y'\}$ and that x is not chosen from $\{x, y'\}$.

Figure O1b provides an example in which the nondegeneracy property is violated. To see this, note that for any $\bar{a}, y \in \Delta$ such that $r(\bar{a}) > r(y)$ and $w(\bar{a}) < w(y)$, we have $g(\bar{a}, w(\bar{a})) > g(y, w(\bar{a}))$, so $\mathcal{P}(\bar{a})$ is empty. In this case, the reference-lowering alternative $\bar{a}$ is also the chosen one, so observing $\{\bar{a}, y\} \succ \{y\}$ does not allow us to tell if the larger menu is preferred because $\bar{a}$ lowers the reference point or because $\bar{a}$ is the preferred choice.

However, the preference illustrated in Figure O1b satisfies the weak nondegeneracy axiom. To see this, note that (i) the reference point at $\{\bar{a}, \bar{b}, c, d\}$, i.e., $w(\bar{a})$, is lower than the reference point at $\{\bar{b}, c, d\}$, i.e., $w(d)$, and (ii) $\mathcal{C}(\{\bar{b}, c, d\}) = \mathcal{C}(\{\bar{a}, \bar{b}, c, d\}) = \{c\}$. In this case, $\bar{a}$ makes the larger menu more desirable even though it is not chosen there, by setting the reference point lower than the reference point at $\{\bar{b}, c, d\}$. Therefore, we have $\bar{a} \succ^* \bar{b}$. This example shows why we cannot confine Definition 1(i) to $A = \{\bar{b}\}$: Even if $r(\bar{a}) > r(\bar{b})$, we may have $\bar{a} \in \mathcal{C}(\{\bar{a}, \bar{b}\})$ ($\bar{a}$ is chosen) or $w(\bar{a}) \geq w(\bar{b})$ ($\bar{a}$ sets a weakly higher reference

point), preventing us from concluding $\bar{a} \succ^* \bar{b}$ with $A = \{\bar{b}\}$. Thus, to conclude $\bar{a} \succ^* \bar{b}$, we may need a larger menu A that contains a "choice fixer" $c \in P_1(\bar{a})$ and a "higher reference setter" $d \in P_2(\bar{a})$. The figure also graphically illustrates Theorem 2, in particular that we observe $\bar{a} \succ^* \bar{b}$ whenever $r(\bar{a}) > r(\bar{b})$, as long as $P_1(\bar{a})$ and $P_2(\bar{a})$ are nonempty, which is a quite weak condition.

The nondegeneracy condition is also a quite weak condition in general, because it holds generically if $\dim(Z) \geq 4$. To see the intuition, note that $\mathcal{P}(x)$ is characterized by three linear inequalities (involving r, w, and $u + \beta w$) and that $\mathcal{S}(x)$ is also characterized by three linear inequalities (involving r, w, and $u + w$), as the above discussion of Figure O1a suggests. Thus, as long as the coefficient matrices (of dimension $3 \times (\dim(Z) - 1)$, because the probabilities must sum to one) have a rank of three, which holds generically, they are nonempty.

Figure O2 illustrates how we can elicit $\bar{a} \succ_r \bar{b}$ when $\bar{a}$ cannot satisfy $\bar{a} \succ^* \bar{b}$ even though data are generated by a PS preference with $r(\bar{a}) > r(\bar{b})$. Figure O2a depicts the indifference curves of the same PS preference as in Figure O1b. However, because $\bar{a}$ is on the boundary of Δ, $P_2(\bar{a})$ is empty, and we cannot establish $\bar{a} \succ^* \bar{b}$ with any $A \ni \bar{b}$. In words, when $\bar{a}$ is a unique reference alternative at $A \cup \{\bar{a}\}$ (i.e., $r(\bar{a}) > r(y)$ for all $y \in A$), the reference point is necessarily higher than that at A, so $A \cup \{\bar{a}\} \succ A$ does not occur as long as $\bar{a}$ is unchosen there.

However, we can still conclude $r(\bar{a}) > r(\bar{b})$ by using some $c \in \text{int}(\Delta)$ such that $r(\bar{a}) > r(c) > r(\bar{b})$, as Figure O2b demonstrates. First, we can elicit $c \succ^* \bar{b}$ by Lemma 18 (note also that $P_1(c)$ and $P_2(c)$ are nonempty). Moreover, we cannot have $c \succ^* \bar{a}$, as c cannot set a reference point at A whenever $\bar{a} \in A$. Thus, we can conclude $\bar{a} \succ_r \bar{b}$ via Definition 1(ii-b).

Figure O1: Nondegeneracy and Weak Nondegeneracy
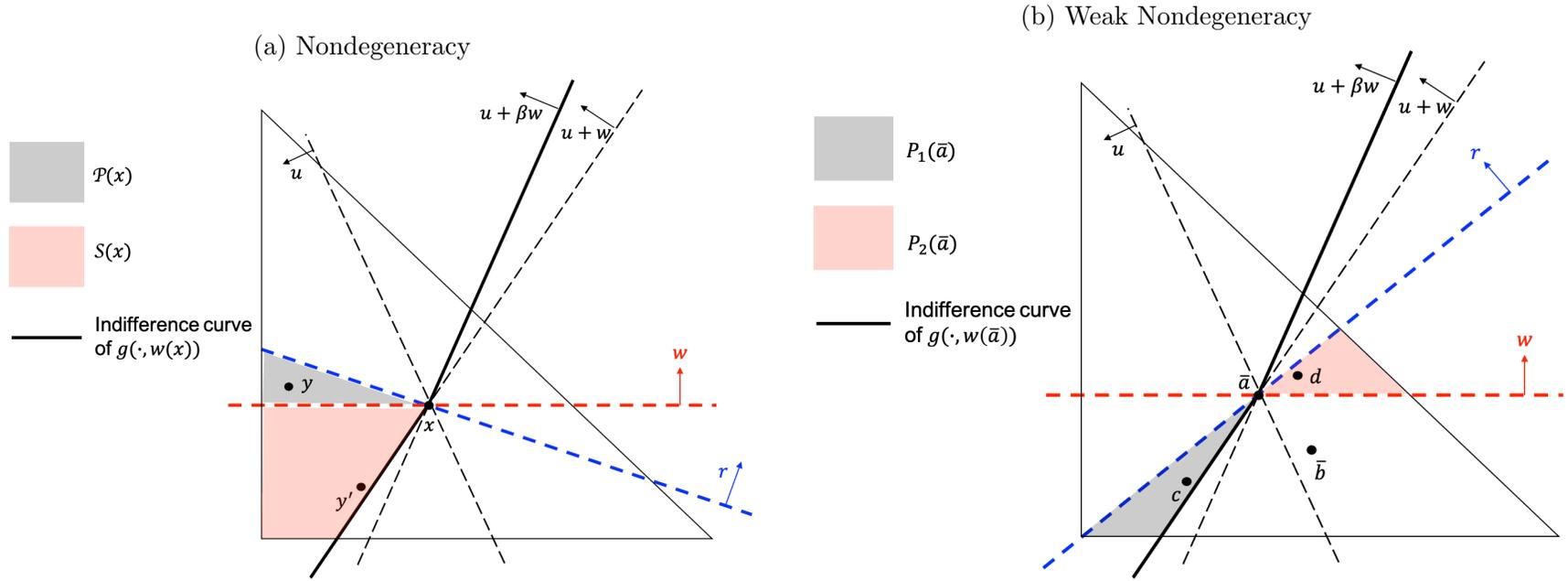
Notes: Panel (a) presents an example of a PS preference that satisfies nondegeneracy. Panel (b) presents an example of a PS preference that satisfies weak nondegeneracy but not nondegeneracy, because $\mathcal{P}(\bar{a})$ is empty for all $\bar{a} \in \Delta$. Each dashed or solid straight line represents an indifference curve of u, w, $u+w$, $u+\beta w$ or r, with an arrow indicating the increasing direction of the utility function. The bold solid line kinked at x in Panel (a) (at $\bar{a}$ in Panel (b)) denotes the indifference curve of the function $g(\cdot, w(x))$ ($g(\cdot, w(\bar{a}))$) defined in Lemma 10. In Panel (a), the black and red shaded areas depict $\mathcal{P}(x)$ and $\mathcal{S}(x)$, respectively, defined in Eq. (4) and (5) in Section 3.1. In Panel (b), the black and red shaded area depicts $P_1(\bar{a})$ and $P_2(\bar{a})$, respectively, defined in Lemma 17. See the text in Appendix O.C for details.

Figure O2: Reference Elicitation on the Boundary

(a) Non-existence of $d \in P_2(\bar{a})$
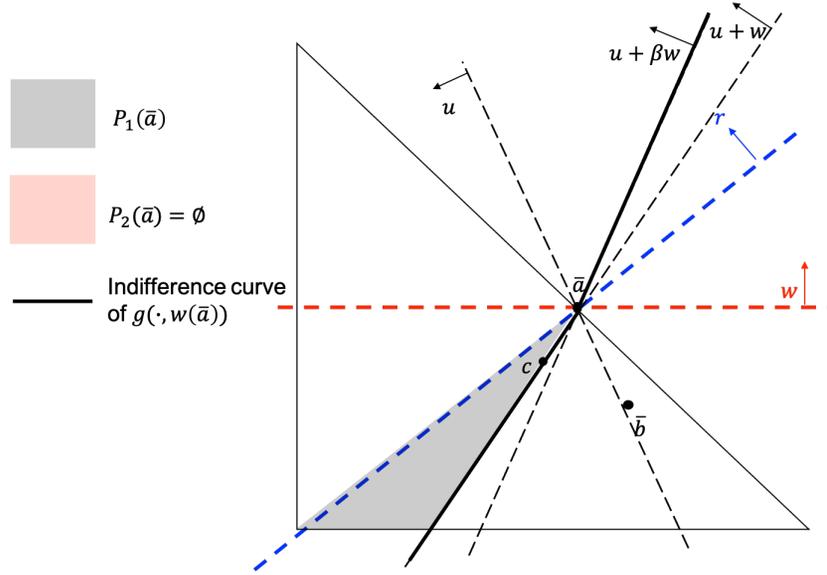
(b) Mediating alternative $c \in \text{int}(\Delta)$
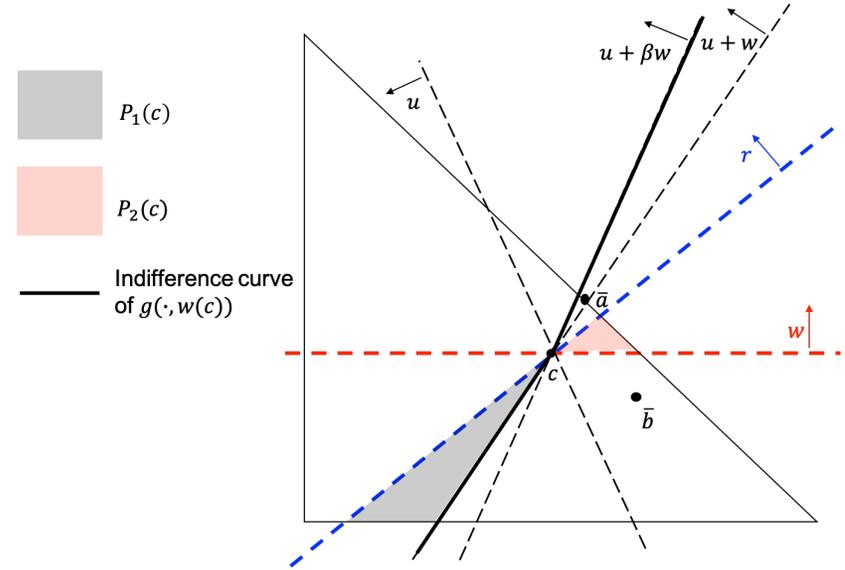
Notes: Panel (a) presents an example of alternatives $\bar{a}, \bar{b} \in \Delta$ such that $r(\bar{a}) > r(\bar{b})$ but $\bar{a} \not\succ^* \bar{b}$. Panel (b) illustrates how we can establish $\bar{a} \succ_r \bar{b}$ via Definition 1(ii-b) using a mediating alternative $c \in \text{int}(\Delta)$. Each dashed or solid straight line represents an indifference curve of u, w, $u + w$, $u + \beta w$ or r, with an arrow indicating the increasing direction of the utility function. The bold solid line kinked at $\bar{a}$ in Panel (a) (at c in Panel (b)) denotes the indifference curve of the function $g(\cdot, w(\bar{a}))$ $(g(\cdot, w(c)))$ defined in Lemma 10. In Panel (a), the black shaded area depicts $P_1(\bar{a})$ defined in Lemma 17. In Panel (b), the black and red shaded areas depict $P_1(c)$ and $P_2(c)$, respectively. See the text in Appendix O.C for details.

References

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